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AUTHOR Capper, Michael R., Comp.
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Abstract

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INSTRUCTIONAL OBJECTIVES FOR A JUNIOR COLLEGE COURSE IN CALCULUS AND ANALYTICAL GEOMETRY

Compiled by

Michael R. Capper

ERIC CLEARINGHOUSE FOR JUNIOR COLLEGES
University of California
Los Angeles, California 90024

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CALCULUS & GEOMETRY OBJECTIVES: SET # 1

UNIT I

FUNDAMENTAL CONCEPTS OF ALGEBRA

An introduction to modern mathematics is presented, followed by discussion of three basic algebraic concepts: sets, relations, and operations. All behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal:** The student will understand the basic elements of set theory.

Objective: 1. The student will define what is meant by $A = B$ where A and B are sets.

100

Sample item: Define what is meant by $A = B$ where A and B are sets.

2. Given a list of pairs of sets, the student will relate the sets, A and B, in each pair by showing whether $A = B$, $A \subseteq B$, $B \subseteq A$, $A \subset B$, or $B \subset A$.

66

Sample item: Write
1 if $A = B$
2 if $A \subseteq B$
3 if $B \subseteq A$
4 if $A \subset B$
5 if $B \subset A$
6 if none of the above are true

a. ___ $A = \{\text{integers}\}$
$B = \{\text{integers greater than zero}\}$

b. ___ $A = \{\text{integral multiples of } 3\}$
$B = \{\text{integral multiples of } 2\}$

Objective: 3. Given two sets, A and B, the student will indicate the elements of $A \cup B$ and $A \cap B$. 80

Sample item: Indicate the union and intersection of the following sets:

a. $A = \{1,3,6\}$
   $B = \{3,5,7\}$

b. A = {negative odd numbers}
   B = {integers divisible by -1}

4. Given two or three sets, the student will draw a Venn diagram representing the sets and given expressions involving the sets. 66

Sample item: If U is the set of integers and $A = \{1,2,3\}$
$B = \{1,2,3,4,5\}$
$C = \{3,4,5,6,7\}$,
draw a Venn diagram representing A,B, and C and cross-hatch neatly the area representing $A \cup (B \cap C)$.

II Goal: The student will understand the concepts of cross-product, relation, and equivalence relation.

Objective: 5. Given two sets, S and T, the student will form $S \times S$, $S \times T$, and $T \times S$. 90

Sample item: Let $S = \{0,1,2\}$ and $T = \{-1,0,1\}$. Form $S \times S$, $S \times T$, and $T \times S$.

6. The student will define terms taken from the following list: ordered pair, product set (cross-product), relation, reflexive, symmetric, transitive, equivalence relation. 100

Sample item: Define the term symmetric in terms of a relation R.

Objective: 7. The student will identify which properties of an equivalence relation are true for a given relation. 66

Sample item: Given a relation $R = \{(a,a),(a,b),(a,c),(b,b),(b,a),(c,c)\}$ on a set $S = \{a,b,c\}$, identify those properties of an equivalence relation which are true for R.

III Goal: The student will understand the terms commutative, associative, and distributive.

Objective: 8. The student will define the terms commutative, associative, and distributive. 100

Sample item: Define commutative with respect to addition of integers.

IV Goal: The student will understand the concept of an operation and be able to apply it to group properties.

Objective: 9. The student will define terms taken from the following list: operation, identity element, inverse of an element. 100

Sample item: Define the term operation.

10. Given an operation, the student will determine whether that operation is commutative, associative, or distributive. 66

Sample item: If $a*b$ means take $a+b+2$ where a and b are integers and $*$ is an operation defined on I, is $*$ commutative? associative? distributive?

11. Given a table of operations, the student will determine whether the set is a group. If the set is not a group, the student will identify those properties which the set lacks to make it a group. 75

Sample item: Given the following tables,

in each case indicate whether or not the set A = {a,b,c} is a group. If the set is not a group, indicate which property or properties are lacking.

a.

o	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

b.

*	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

UNIT II

THE REAL NUMBER SYSTEM

This unit discusses properties of the real numbers which are prerquisite for subsequent areas of study. Unless otherwise noted, all behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal**: The student will be able to demonstrate that a given set is a field.

Objective: 1. At home with open books permitted, the student will show that the set of rational numbers, Q, defined as ordered pairs, is a field by proving all the field properties for Q. 95

II **Goal**: The student will be able to utilize the rules governing inequalities and absolute values for real numbers.

Objective: 2. The student will indicate whether the given mathematical sentences are true or false and then either prove or give a counterexample, respectively. 90

Sample item: Indicate whether the following is true or false; if true, prove and if false, give a counterexample; If $a<b<0$, then $b^2<a^2$.

Objective: 3. Given an expression, the student will indicate the absolute value of that expression. 90

Sample item: Evaluate the following:

a. $|3-5|$

b. $\left|\frac{4-x}{y}\right|$ if $y>0>x$

III **Goal:** The students will understand the representation of the real numbers by a number line.

Objective: 4. The student will define terms from the following list using set notation: open interval, closed interval, half-open interval. 100

Sample item: Define the term open interval using set notation and indicate the symbol for the open interval from -3 to 8.

Objective: 5. The student will prove given statements concerning the concept of length on the real line. 80

Sample item: Show that $|b-a|<|a-c|$ if $a<b<c$.

Objective: 6. Given a set of real numbers, the student will sketch its graph on the real number line. 90

Sample item: Sketch the graph of the following sets:

a. $\{x \mid |x|<1\}$

b. $\{n \mid n \in N\}$

UNIT III

ALGEBRAIC EXPRESSIONS

Proficiency in manipulative skills is necessary for the application of mathematical principles. This unit discusses types of algebraic expressions and the manipulation of such expressions. All behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal:** The student will gain proficiency in manipulating expressions involving integral and rational exponents and radicals.

Objective: 1. The student will simplify a list of expressions involving integral exponents. 70

Sample item: Simplify the following and remove all negative exponents:

a. $\frac{1}{x}\left(\frac{1}{y}\right)^{-1}\left(\frac{x}{y}\right)^{3}$ b. $\frac{(stu)^{-3}}{(st)^{2}u^{-4}}$

Objective: 2. The student will simplify a list of expressions involving rational exponents. 70

Sample item: Simplify the following and remove all negative exponents:

a. $\left((a^{14})^{\frac{1}{2}}\right)^{\frac{1}{4}}$ b. $\left(\frac{a^{10}b^{-3}}{b^{15}}\right)^{\frac{1}{2}}$

Objective: 3. The student will simplify a list of expressions involving radicals and rational exponents. 70

Sample item: Simplify the following:

a. $\sqrt[3]{4x^6(16x^8)^{\frac{1}{4}}}$ b. $\sqrt[3]{\sqrt{x^{12}y^{-3}}}$

II Goal: The student will be able to perform operations with and factor polynomials.

Objective: 4. The student will perform the indicated operation and find the degree of the resulting polynomial. 70

Sample item: Perform the indicated operation and find the degree of the resulting polynomial:

a. $(x^3+3x+2) - (x^2-3x+5)$

b. $\frac{6x^2 + 4x}{2x}$

Objective: 5. The student will factor each of the given polynomials by completing the square. 75

Sample item: Factor by completing the square:

a. $16x^2 + 64x - 80$ b. $a^2 + 8a + 1$

Objective: 6. The student will factor each of the given polynomials by any method. 75

Sample item: Factor by any method:

a. $16x^4 - y^8$ b. $x^3 + x^2 + x + 1$

Objective: 7. The student will perform the given operation and simplify the given rational integral algebraic expressions. 70

Sample item: Simplify the following:

a. $1 + \frac{1}{1 + \frac{1}{x}}$ b. $\frac{b}{a^{\frac{1}{2}}} + \frac{a}{b^{-\frac{1}{2}}} + \sqrt{\frac{b}{a}}$

UNIT IV

EQUATIONS AND INEQUALITIES

The solving of equations and inequalities is one of the more important tasks in the study of elementary algebra. Many applications of mathematics to other fields are based on the concept of finding or interpreting a single variable in terms of other known parameters. All behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal:** The student will be able to solve linear equations and applications involving linear equations.

Objective: 1. The student will solve a list of linear equations in one variable. 85

Sample item: Give the solution set for each of the following equations:
a. $6x + 3 = 4$
b. $2/(4-x) = 1/(2x-8)$

Objective: 2. Given a list of formulas, the student will solve each for the indicated variable in terms of the other variables. 85

Sample item: Solve for the indicated variable in terms of the remaining variables in the following equations:
a. $p = 2(l + w)$ for l
b. $F = (9/5)C + 32$ for C

Objective: 3. Given an application (word problem), the student will translate the problem into mathematical symbols involving a linear equation.

66

Sample item: Translate the following problems into mathematical symbols. Do not solve the problems.

a. If a car travels 18 miles on a gallon of gas, how far will a \$1.00 purchase take him if one gallon costs 32¢?

Objective: 4. Given an application, the student will solve for the unknown variable.

60

Sample item: Solve the following problem: The sum of the digits of a 2-digit number is 13, the difference equals 5. What is the number?

II Goal: The student will be able to use the quadratic formula to solve quadratic equations and applications involving quadratic equations.

Objective: 5. The student will indicate for a given list of quadratic equations whether each equation has a double root, two real solutions, or no real solutions.

90

Sample item: Indicate whether the following equations have a double root, two unique real roots, or no real roots:

a. $2x^2 + 6x - 3 = 0$

b. $x = \frac{1}{1-x}$

Objective: 6. Given a list of quadratic equations, the student will solve each by using the quadratic formula.

85

Sample item: Solve the following by using the quadratic formula:

a. $x^2 + 6x - 7 = 0$

b. $x^2 - 8 = 0$

Objective: 7. Given applications involving a quadratic equation, the student will solve each by using the quadratic formula.

60

Sample item: The product of two consecutive integers is 8. Show that no solution is possible.

III **Goal:** The student will be able to solve miscellaneous equations in one variable by exponential manipulations or transformations.

Objective: 8. Given a list of equations in one variable, the student will find the solution set of each by using a suitable exponential manipulation or transformation.

70

Sample item: Give the solution set for each of the following:

a. $\sqrt{2x+3} - \sqrt{x-3} = 3$

b. $\sqrt[3]{\sqrt{x+2}} = 2$

c. $y^4 - 12y^2 + 32 = 0$

IV **Goal:** The student will be able to solve inequalities in one variable and graph the solution set.

Objective: 9. Given a list of linear inequalities, the student will solve each and sketch the graph of the solution set.

80

Sample item: Indicate and graph the solution set to each of the following:

a. $2x + 3 \leq 7$

b. $3(x+4) \geq 7(5-x)$

Objective: 10: Given a list of linear inequalities involving absolute values, the student will solve each and sketch the graph of the solution set.

80

Sample item: Indicate and graph the solution set to each of the following:

a. $|x+2| \leq 3$

b. $|3-x| > 2$

Objective: 11: Given a list of quadratic inequalities, the student will solve each and sketch the graph of the solution set.

80

Sample item: Give the solution set and sketch the graph of the solution set for each of the following:

a. $x^2 + x - 6 > 0$

b. $x^2 - 7x + 6 < 0$

Objective: 12: Given a list of inequalities involving rational integral algebraic expressions, the student will solve each and sketch the graph of the solution set.

80

Sample item: Give the solution set and sketch the graph of the solution set for each of the following:

a. $\frac{1}{x} < 2$

b. $\frac{x+1}{x+3} \geq 4$

UNIT V

FUNCTIONS AND GRAPHS

Correspondences between sets and mathematical pictures that represent these correspondences are important concepts which tie classroom mathematics to the world outside the closed educational environment. Basic properties of functions and graphs are studied in this unit. All behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal**: The student will understand the following terms: function, domain, range, even and odd function, independent variable, dependent variable.

Objective: 1. The student will define terms from the above list.

100

Sample item: Define the term function.

Objective: 2. The student will give the domain and range of a given list of functions.

85

Sample item: Give the domain and range of the following functions:

a. $f(x) = |1-x|$
b. $g(x) = \sqrt{1-x}$
c. $h(x) = \frac{1}{1-x}$

Objective: 3. The student will distinguish even and odd functions.

80

Sample item: Write
E if the function is even
O if the function is odd
N if the function is neither even or odd:

a. $f(x) = x^2 - x^4$
b. $g(x) = \frac{1}{x+1}$

II **Goal**: The student will be able to graph functions of one and two variables.

Objective: 4. The students will define terms from the following list: graph, increasing, decreasing, strictly increasing, strictly decreasing, maximum (minimum) value, excluded region. 100

Sample item: Define what it means for a function to be strictly increasing and give an example of one.

Objective: 5. The student will graph a given list of relations and sets. 75

Sample item: Graph the following:
a. $S = \{(x,y): y>x\}$
b. $S = \{(x,y): |x| = 2\}$

Objective: 6. The student will sketch the graphs of a given list of functions of one variable. 80

Sample item: Sketch the graphs of the following:
a. $f(x) = |x|$
b. $f(x) = 3x + 7$

Objective: 7. The student will sketch the graphs of a given list of functions of two variables. 70

Sample item: Sketch the graphs of the following equations:
a. $xy + y = 1$
b. $2xy = x^2 + x$

III **Goal**: The student will understand the equation of the circle and the distance formula.

Objective: 8. The student will use the distance formula to show given points are vertices of certain geometrical figures. 70

Sample item: Show that the points (1,2), (7,2), and (4,9) are vertices of an isosceles triangle.

Objective: 9. The student will find the equation of a circle given certain conditions. 70

Sample item: Find the equation of the circle with center at (3,3) and tangent to both axes.

Objective: 10. Given the equation of a circle, the student will find the center and radius of the circle. 70

Sample item: Find the center and radius of the circle given by the equation

$x^2 + y^2 + 4x + 2 = 0$.

IV **Goal:** The student will be able to graph quadratic equations.

Objective: 11. The student will graph a given quadratic equation and find the x-intercepts, y-intercepts, and vertex. 70

Sample item: Graph the equation $y = (x+2)^2$ and give the x-intercepts, y-intercepts, and vertex. State whether the vertex is a maximum, minimum, or neither.

V **Goal:** The student will understand the concept of variation.

Objective: 12. The student will define terms taken from the following list: constant of variation, directly proportional, inversely proportional. 100

Sample item: Define what it means for one variable to vary inversely proportional to another.

Objective: 13. The student will solve applications of problems involving variation. 66

Sample item: If the area of a circle varies directly proportional to the square of the radius, what is the constant of proportionality?

VI Goal: The student will understand the concepts of composite and inverse functions.

Objective: 14. Given two functions, f and g, the student will form f g and g f, and indicate whether or not $g = f^{-1}$.

70

Sample item: If $f(x) = 2x$ and $g(x) = x + 1$, find f g and g f; does $g = f^{-1}$?

Objective: 15: Given a function f, the student will find the inverse function of f.

66

Sample item: Find the inverse function of $f(x) = 3x - 1$ and specify the domain and range for which $f^{-1}(f(x)) = x$.

UNIT VI

SYSTEMS OF EQUATIONS AND INEQUALITIES

The concept of a solution for a given equation can be generalized to finding the solution set for more than one equation in more than one unknown. This unit investigates methods of solving systems of equations and also discusses graphing techniques with respect to the solution of a system of inequalities. Unless otherwise noted, all behaviors will be performed under test conditions: in class, without open books or notes, and all work is to be shown without scratch paper.

I **Goal**: The student will be able to apply the substitution method to solve systems of equations.

Objective: 1. The student will use the method of substitution to find a solution set for given systems of equations.

Sample item: Find the solution set for the following system: 75

$$x^2 + y = 10$$
$$x + y = 4$$

Objective: 2. The student will show that if p and q are two equations in x and y, if $S_p = \{(x_1,y_1)\}$, $S_q = \{(x_2,y_2)\}$ are the solution sets to p and q respectively, and if S is the solution set to the system $\{p,q\}$, then $S = S_p \cap S_q$.

II **Goal**: The student will be able to solve systems of linear equations in more than two variables.

Objective: 3. The student will solve a system of two equations in two unknowns by the reduction of simultaneous equations. 90

Sample item: Solve

$$x + y = 10$$
$$x - y = 6$$

by reducing the simultaneous equations.

Objective: 4. The student will find the solution set of a system of n linear equations in m unknowns by reducing simultaneous equations or by the substitution method, where $n = 2,3,4$, and $m = 2,3,4$. 70

Sample item: Give the solution set of the following system:

$$\begin{aligned} x + y + z &= 2 \\ 2x + 3y - z &= 9 \\ x + z &= 0 \end{aligned}$$

Objective: 5. The student will use matrix methods to find the solution set of a system of linear equations. 70

Sample item: Use matrix methods to solve

$$\begin{aligned} x - y + z &= -10 \\ x + 2y - 3z &= 27 \\ 2x + y - 2z &= 23 \end{aligned}$$

III **Goal**: The student will understand the following concepts and be able to apply them to solve systems of linear equations: determinant, minor, cofactor, and Cramer's rule.

Objective: 6. The student will find the determinants of a given list of matrices by expanding by minors. 75

Sample item: Find det A by expanding by minors:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

Objective: 7. The student will show various properties of determinants by evaluating determinants in equations or expressions. 75

Sample item: If

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix},$$

show that det A = det A.

Objective: 8. The student will use Cramer's rule to solve a given system of linear equations. 80

Sample item: Use Cramer's rule to solve

$$\begin{aligned} x + 2y + 3z &= 2 \\ 2x + 4y + 2z &= -1 \\ x + 2y - 2z &= 5. \end{aligned}$$

IV Goal: The student will be able to solve a system of inequalities.

Objective: 9. Given a system of inequalities, not necessarily linear, the student will indicate the solution set by sketching the graph of the system. 80

Sample item: Indicate the solution set of the following system by sketching the graph of the inequalities and shading heavily the solution set:

$$\begin{aligned} x + y &< 2 \\ x^2 + y^2 &\leq 25 \end{aligned}$$

Objective: 10. At home with open books and notes the student will solve a linear programming application of solving systems of linear inequalities. 50

UNIT VII

POLYNOMIALS

The theory of polynomials is generalized beyond the quadratic to a polynomial of arbitrary degree, and factorization and root extraction is discussed for an arbitrary polynomial. All behaviors will be performed under test conditions: in class, without open book or notes, and all work is to be shown without scratch paper.

I **Goal:** The student will demonstrate the algebraic properties of polynomials.

Objective: 1. Given two polynomials, $f(x)$ and $g(x)$, the student will form and give the degree of $f(x) + g(x)$ and $f(x)g(x)$. 85

Sample item: If $f(x) = x^2 + 2x + 3$ and $g(x) = x - 1$, form $f(x) + g(x)$ and $f(x)g(x)$ and give the degree of each.

Objective: 2. The student will prove algebraic properties which hold for F x . 66

Sample items:
a. Show $F[x]$ is associative with respect to both addition and multiplication.
b. Show that $f(x) = x^2$ has no multiplicative inverse in $F[x]$.

II **Goal:** The student will understand and be able to apply properties of division to $F[x]$.

Objective: 3. The student will form quotients of polynomials by long division and synthetic division. 80

Sample item: If $f(x) = x^3 - 3x^2 + 4$ and $g(x) = x - 3$, find $f(x)/g(x)$ by both long division and synthetic division.

Objective: 4. The student will use the remainder and factor theorems to show whether f(c), for some given c, is a root of f(x). 80

Sample item: If $f(x) = x^3 - 3x^2 + 4$, is $x = 2$ a root? Prove your result, i.e. don't just substitute into the equation.

III Goal: The student will understand principles of factorization theory.

Objective: 5. Given roots of a polynomial, the student will exhibit that polynomial. 80

Sample item: Find the polynomial whose roots are 1-i and 1+i.

Objective: 6. The student will find the zeroes of a given polynomial and state the multiplicity of each root. 75

Sample item: Find the zeroes of $f(x) = x^4 - 1$ and state the multiplicity of each root.

Objective: 7. The student will find rational roots of a polynomial with integral coefficients. 75

Sample item: Find the roots of $6x^2 - x - 1$.

IV Goal: The student will understand the basic concepts of complex numbers.

Objective: 8. The student will be able to add, subtract, multiply, and divide complex numbers. 90

Sample item: Evaluate (1+i)/(1-i).

Objective: 9. The student will be able to show any of the following properties:

a. $\overline{z+w} = \bar{z}+\bar{w}$
b. $\overline{zw} = \bar{z}\,\bar{w}$
c. $\bar{z} = z$ iff z is real
d. $\overline{z^n} = \bar{z}^n$ for all n N 90

Objective: 10. The student will show that if $f(z) = 0$, then $f(\bar{z}) = 0$. 75

CALCULUS & GEOMETRY OBJECTIVES: SET # 2

UNIT 0: REVIEW OF PREREQUISITE MATHEMATICS

This unit will consider inequalities, set theory as applied to intervals, absolute values, coordinate systems, the distance formula and a review of trigonometry.

Objectives

I. Goal: The student will know the definitions of $<$, $>$, $\leq$, $\geq$ and be able to give geometric interpretations of them.

Objective: 1. Taking "positive number" as undefined, the student will, in five minutes, with no references, define with 100% accuracy the above symbols.

2. Given five simultaneous linear inequalities, the student will, in two minutes, with no references, graph the solutions of at least four on number lines (i.e., "x-axes"), each with 100% accuracy.
Sample Test Item: (Closed book, one minute.) Graph all solutions of $-7 < x \leq 1$.

II. Goal: The student will be able to solve polynomial and other rational inequalities.

Objective: 3. Given a factored polynomial, the student will, in five minutes, with no references, draw a schematic graph (indicating all zeros and all points at which the polynomial is positive).
Sample Test Item: (Closed book, five minutes.) Graph schematically: $(x - 2)^2(x + 3)^5(x + 6) > 0$.

4. Given a polynomial or other rational inequality, the student will, in five minutes, with no references, solve algebraically for all solutions, with 80% accuracy.
Sample Test Items: (Closed book, five minutes each.) Solve (a) $2x + 3 < 5 - x \leq 4x + 15$,

(b) $$\frac{(x - 2)^2(x + 3)^5}{(x + 6)^3} > 0.$$

III. Goal: The student will know basic terminology of set theory and be able to apply it to intervals.

Objective: 5. Given $A = \{x: x \leq -2\}$, $B = \{x: -10 < x < 5\}$, the student will, in five minutes, with no references, (a) define A in words, (b) define A B in either words or symbols, (c) define A B in either words or symbols, with 80% accuracy.

6. In set theory symbols, the student will, in five minutes, with no references, define (a,b), $[a,b]$, $[a,b)$, $(a,b]$, (a,∞), $(-\infty,a)$, $[a,\infty)$, $(-\infty,a]$, $(-\infty,\infty)$, as they apply to intervals, with 80% accuracy.

IV. **Goal:** The student will know and be able to apply the definition of the absolute value of a number.

Objective: 7. The student will, in one minute, with no references, be able to define $|a|$, with 100% accuracy.

8. Given an equation or inequality involving absolute values, the student will, in one minute, with no references, rewrite equivalent inequalities (or equations) which do not involve absolute values, with 100% accuracy.
Sample Test Item: (Closed book, one minute.)
Rewrite $|x - 3| < 2$ without using absolute values.

9. Given an inequality involving absolute values, the student will, in one minute, with no references, indicate all solutions on a number line.
Sample Test Item: (Closed book, one minute.)
Graph all of the solutions of $|x - 3| < 2$.

V. **Goal:** The student will be able to validate the use of one- and two-dimensional rectangular coordinate systems.

Objective: 10. The student will, in five minutes, with no references, explain in 50 words or less why the x-axis is a model for the real numbers and why the xy-plane is a model for the set of all real number pairs, with 75% accuracy.

VI. **Goal:** The student will be able to graph efficiently equations wherein y is expressed in terms of x.

Objective: 11. Given an equation with y expressed in terms of x, the student will determine, in five minutes, with no references, for what values of x y is not defined, with 80% accuracy.
Sample Test Item: (Closed book, five minutes.)
For what values of x is y not defined? $y = \sqrt{4 - x}$.

12. Given an equation with y expressed in terms of x, the student will, in five minutes, with no references, determine the x- and y-intercepts of the graph, with 80% accuracy.
Sample Test Item: (Closed book, five minutes.)

Find the x- and y-intercepts of $y = \sqrt{4 - x^2}$.

Objective: 13. Given an equation with y expressed in terms of x, the student will, in two minutes, with no references, determine whether or not there is symmetry about the y-axis, with 100% accuracy.
Sample Test Item: (Closed book, two minutes.)
Is the graph of $y = \sqrt{4 - x^2}$ symmetric about the y-axis?

VII. Goal: The student will know and be able to both derive and use the distance formula.

Objective: 14. The student will, in five minutes, with no references, state with 100% accuracy and derive with 80% accuracy the distance formula.

15. The student will, in five minutes, with no references, compute the distances between three pairs of points in the xy-plane, with 80% accuracy.
Sample Test Item: (Closed book, five minutes.)
Find the distance (a) between (-5,2) and (-3,-6), (b) between (14,-7) and (x,y), and (c) between (cos A, sin A) and (cos B, sin B).

VIII. Goal: The student will be able to characterize given circles with equations.

Objective: 16. Given a circle, the student will, in two minutes, with no references, determine an equation characterizing it, with accuracy in all detail, except possible arithmetic errors.
Sample Test Item: (Closed book, two minutes.)
Write an equation of the circle with center (-3,7) and radius 4.

17. Given an equation of a circle, the student will, in five minutes, with no references, determine the center and radius, with no errors other than in arithmetic.
Sample Test Item: (Closed book, five minutes.)
Find the center and radius of the circle determined by $x^2 + y^2 - 4x + 6y - 3 = 0$.

IX. Goal: The student will be able to define trigonometric functions in terms of coordinates and give analytic derivations of basic identities.

Objective: 18. The student will, in five minutes, with no references, define five of the trigonometric functions, with at least 80% accuracy.

19. The student will, in 20 minutes at most, with no references, derive one of the standard trigonometric identities, with 80% accuracy.
Sample Test Item: (Closed book, 20 minutes.)
Derive the formula for cos(A - B).

UNIT 1: FUNCTIONS

This unit will consider definition of "relation," the definition of "function," types of functions, graphs of functions, techniques of graphing, combinations of functions and their graphs.

Objectives

I. **Goal:** The student will know what a "relation" is, in the mathematical sense.

Objective: 1. The student will, in three minutes, with no references, define "relation," with 100% accuracy.

2. The student will, in two minutes, with no references, give an example of a relation, with 100% accuracy.

II. **Goal:** The student will know and be able to apply the definition of "function."

Objective: 3. The student will, in three minutes, with no references, define "function," with 100% accuracy.

4. Given a list of five functions, the student will, in five minutes, with no references, determine whether or not each is a function, with at least four correct determinations.

III. **Goal:** The student will know and be able to apply the definitions of "constant," "polynomial," "rational," "algebraic," and "transcendental," as applied to functions.

Objective: 5. The student will, in five minutes, with no references, define four of the above listed terms.

6. In five minutes, with no references, the student will give examples of each of the following, with 100% accuracy.

(a) A constant function.

(b) A nonconstant polynomial.

(c) A rational function which is not a polynomial.

(d) An algebraic function which is not a rational function.

IV. <u>Goal:</u> The student will be able to graph and analyze algebraic functions.

<u>Objective:</u> 7. Given four linear functions, the student will, in five minutes and with no references, determine the slope and y-intercept of at least three.
<u>Sample Test Item:</u> (Closed book, five minutes.)
Find the slope and y-intercept of each of at least three of the following:

(a) $y = 3x - 16$; (b) $\frac{y - 2}{x - 3} = -4$;

(c) $y = 5$; (d) $\frac{x}{2} - \frac{y}{3} = 1$.

8. Given five linear functions, the student will, in five minutes, with no references, graph at least four correctly.
<u>Sample Test Item:</u> (Closed book, five minutes.)
Graph the following:

(a) $2x - 5y - 10 = 0$; (b) [illegible] 9;

(c) $8 - 2y = 3$; (d) $\frac{x}{3} - \frac{y}{2} = 1$;

(e) $\frac{x - 4}{y - 2} = 12$.

9. Given five linear functions, the student will, in five minutes, with no references, select (a) two that have parallel graphs, (b) two that have perpendicular graphs, both with 100% accuracy.
<u>Sample Test Item:</u> (Closed book, five minutes.)
Select from the following linear functions two that have

(a) parallel graphs, (b) perpendicular graphs:

(i) $y = 3x - 6$; (ii) $2x - 6y - 3 = 0$;

(iii) $5x = 2y - 1$; (iv) $6x - 2y = 9$;

(v) $x - 3y - 3 = 0$.

10. Given descriptions of five lines, the student will, in 15 minutes, with no references, determine the equations of at least four.
<u>Sample Test Item:</u> (Closed book, 15 minutes.)
Find equations of at least four of the following lines:

(a) through $(-2, 3)$ and $(7, 8)$;

(b) through (0, 9) and of slope $\frac{1}{2}$;

(c) through (-4, 2) and parallel to $y = 3x - 4$;

(d) through (3, - 1) and perpendicular to $y = x - 5$;

(e) with x- and y- intercepts 3 and - 4, respectively.

11. Given a polynomial function, the student will discern at sight, with no references, the general configuration of its graph (whether its shape is basically that of a U or that of an S, and whether the U or S is pointing up or down).
<u>Sample Test Item:</u> (Closed book, two minutes.)
Sketch the general configuration of the graphs of each of the following:

(a) $y = -2x^2 +$ (lower powers of x);

(b) $y = 5x^5 + \ldots\ldots$; (c) $y = \frac{1}{2}x^{10} - \ldots\ldots$;

(d) $y = -1000x^{15} - \ldots\ldots$; (e) $y = \frac{1}{1000}x^{22} - \ldots\ldots$

12. Given a rational function, the student will, in 10 minutes, with no references, graph it, noting any asymptotes, intercepts and symmetry, as well as domain of definition, with at least 80% accuracy.
<u>Sample Test Item:</u> (Closed book, 10 minutes.)
Graph the following, noting any asymptotes, intercepts, symmetry, as well as the domain of definition:

$$y = \frac{x(x^2 - 9)}{(x^4 - 16)(x^2 - 1)}$$

V. <u>Goal:</u> The student will know the definitions of various combinations of functions and their implications on graphing.

<u>Objective:</u> 13. Given functions f: R R and g: R R, the student will, in five minutes, with no references, define

(a) fg, (b) f - g, (c) f/g, (d) $f_\circ g$, with 90% accuracy.

14. Given three combinations of functions, the student will, in 10 minutes, with no references, graph at least two correctly.
<u>Sample Test Item:</u> (Closed book, 10 minutes.)
Correctly graph at least two of the following, given the graph of y sin x:

(a) $y = x - \sin x$; (b) $y = \sin\left(\frac{1}{x}\right)$;

(c) $y = x \cdot \sin x$.

VI. <u>Goal:</u> The student will be able to define, recognize, graph and determine conic sections (i.e., quadratic relations).

<u>Objective:</u> 15. Given a particular conic section, the student will, in three minutes, with no references, define it with 100% accuracy.

16. Given five quadratic relations, the student will, in five minutes, with no references, identify what kind of conic section each is, with 80% accuracy.
<u>Sample Test Item:</u> (Closed book, five minutes.)
Identify each of the following conics:

(a) $\frac{x^2}{16} - \frac{y^2}{25} = 1$; (b) $(y - 3)^2 = - 12(x - 6)$;

(c) $(y - 5)^2 - (x - 3)^2 = 16$;

(d) $\frac{x^2}{16} - \frac{(y - 1)^2}{64} = 1$; (e) $x^2 - y^2 = 0$.

17. Given five quadratic relations, the student will, in 20 minutes, with no references, graph at least four correctly.
<u>Sample Test Item:</u> (Closed book, 20 minutes.)
Graph at least four of the following:

(a) $25x^2 - 16y^2 = 144$;

(b) $y^2 - 12x - 6y - 63 = 0$;

(c) $(y - 5)^2 - (x - 3)^2 = 16$;

(d) $\frac{x^2}{16} - \frac{(y - 1)^2}{64} = 1$;

(e) $x^2 - y^2 = 0$.

18. Given three nondegenerate conic sections, with foci and vertices (or equivalent conditions) specified, the student will, in 15 minutes, with no references, write equations for two, with not more than one arithmetic error in each equation.
<u>Sample Test Item:</u> (Closed book, 15 minutes.)
Derive equations for at least two of the following:

(a) the parabola with focus $(-2, 4)$, directrix $y = x - 2$;

(b) the ellipse with foci $(2, - 4)$, vertices $(2, - 5)$;

(c) the hyperbola with asymptotes $2y = \pm x$, vertices $(- 5, 0)$.

UNIT 2: LIMITS AND CONTINUITY

This unit will consider the definitions of both finite and infinite limits, theorems on limits of sums, products and quotients, the definition of continuity and its graphical interpretation.

Objectives

I. **Goal:** The student will know and be able to apply the definition of "limit," both finite and infinite.

Objective: 1. The student will, in three minutes, with no references, define "finite limit," with 80% accuracy.

2. The student will, in three minutes, with no references, define "infinite limit," with 80% accuracy.

3. Given a function that "obviously approaches at certain limit at a certain point," the student will, in 10 minutes, with no references, prove that the function approaches the limit, using $\delta - \epsilon$ technique, with 70% accuracy.
Sample Test Item: (Closed book, 10 minutes.)
Using δ and ϵ, prove that $\lim_{x \to 2} x^2 = 4$.

4. The student will, in two minutes, with no references, give an example of a function that does not have a limit at some point, with 100% accuracy.

II. **Goal:** The student will be able to state, prove and use theorems on limits of sums, products and quotients.

Objective: 5. The student will, in two minutes, with no referen ces, state a specified limit theorem with 90% accuracy.
Sample Test Item: (Closed book, two minutes.)
State the theorem concerning the limit of a sum of two functions.

6. The student will, in 10 minutes, with no references, prove a specified limit theorem, with 75% accuracy.

7. Given a mathematical statement involving limits, the student will in one minute, with no references, cite the limit theorem which justifies it. 100% accuracy.
Sample Test Item: (Closed book, one minute.)

If $x \to 2$ and $y \to 3$ as $t \to 16$, why does $xy \to 6$ as $t \to 16$?

III. Goal: The student will be able to define and interpret graphically "continuity."

Objective: 8. The student will, in two minutes, with no references, define "f is continuous at x ," with 100% accuracy.

9. Given a function f and a point x , the student will, in 10 minutes, with no references, prove (using $\delta - \epsilon$) that f is continuous at x , with 80% accuracy.
Sample Test Item: (Closed book, 10 minutes.)
Prove: $f(x) = 2x - 3$ is continuous at $x = 2$.

10. Given a function f undefined at a certain point x , continuous at all other points, the student will, in 10 minutes, with no references, determine how to define f at x so that f is continuous at x . If he can't carry out the computations, he will at least outline a method for making the determination.
Sample Test Item: (Closed book, 10 minutes.)
The function defined by the following equation is continuous for all real x except x 2, at which point it is undefined. Determine a definition of the function at x 2 which renders the function continuous for all x. $y = \frac{x^3 - 8}{x-2}$

11. The student will, in two minutes, with no references, sketch a discontinuous function, indicating an for which no appropriate can be found. 100% accuracy.

UNIT 3: DERIVATIVES

This unit considers the definition and application of "derivative" (applications to tangent lines, extrema, velocity), continuity of a differentiable function, differentiation formulas for algebraic functions, implicit differentiation, higher derivatives, partial differentiation and anti-differentiation.

Objectives

I. Goal: The student will be able to define, derive, compute and apply the definition of derivative.

Objective: 1. The student will, in two minutes, with no references, define "the derivative of f at x ," with 90% accuracy.

2. Given a list of 10 elementary algebraic functions, the student will, in five minutes, with no references, compute the derivative of each, with at least 90% accuracy.
Sample Test Item: (Closed book, five minutes.)
Compute the derivatives of at least nine of the following:

(a) $y = x^2$; (b) $y = 3$; (c) $y = \frac{1}{x}$;

(d) $y = \frac{x^2}{x-3}$; (e) $y = (x - 2)^{16}$;

(f) $y = (x^3 - 16x^2 - 1)(x^5 - 13x - 2)$;

(g) $y = \sqrt{x}$; (h) $y = \sqrt{3x^2}$;

(i) $y = 0$; (j) $y = \sqrt[3]{x}$.

3. Given a basic theorem on differentiation, the student will, in 10 minutes, with no references, prove it, with 75% accuracy.
Sample Test Item: (Closed book, 10 minutes.)
Prove: $\frac{d(uv)}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$.

4. Given a problem involving tangent lines, extrema, or velocity, the student will, in 10 minutes, with no references, outline a method of proof, with 100% accuracy.

Sample Test Item: (Closed book, 10 minutes.) Considering it as a minimum problem, derive a formula for the distance from the point (x, y) to the line determined by $Ax + By + C = 0$.

5. Given a problem of the type in 4, the student will, in 10 minutes, with no references, carry out the computations, with no errors other than one or two in arithmetic.

II. Goal: The student will know that a function differentiable at a point is also continuous at that point.

Objective: 6. The student will, in 10 minutes, with no references, prove that a function differentiable at a point is also continuous at that point, with 80% accuracy.

7. The student will, in 2 minutes, with no references, sketch the graph of a function that is continuous, but not differentiable, with 100% accuracy.

III. Goal: The student will be able to differentiate implicitly.

Objective: 8. Given an implicitly defined algebraic function, the student will, in five minutes, with no references, and with at most two errors in arithmetic and/or algebra, compute the derivative of the function at a specific point.

Sample Test Item: (Closed book, five minutes.) Find y' if $xy^2 - x^2y - x^3 = 6$.

IV. Goal: The student will know how to compute higher derivatives and apply them to graph analysis.

Objective: 9. Given a function, the student will, in five minutes, with no references and no errors other than in arithmetic, compute the third derivative.

Sample Test Item: (Closed book, five minutes.) Compute the third derivative of $y = 16x^2 - \frac{1}{x}$.

10. Given a function and a point, the student will, in 10 minutes, with no references, prove that the graph is concave downward at the point, with 90% accuracy.

Sample Test Item: (Closed book, 10 minutes.) Prove: $y = \sqrt{x}$ is concave downward at $x = 1$.

V. Goal: The student will know the notation of partial differentiation and be able to compute simple partial derivatives.

Objective: 11. Given a function of two variables, the student will, in five minutes, with no references, compute the partial derivative with respect to one of the variables, with 90% accuracy.

Sample Test Item: (Closed book, five minutes.)

If $z = x^2y - \frac{x}{y}$, compute $\frac{\partial z}{\partial y}$.

VI. <u>Goal:</u> The student will know what an antiderivative is and be able to compute one, as well as apply antiderivatives to motion problems.

<u>Objective:</u> 12. Given a polynomial function, the student will, in two minutes, with no references, compute an antiderivative, with 90% accuracy.
<u>Sample Test Item:</u> (Closed book, two minutes.)
Compute an antiderivative of $y = x^5 - 4x^3 + x^2 - 6x - 1$.

13. Given a motion problem, the student will, in five minutes, with no references, outline a method of solution, with 100% accuracy.
<u>Sample Test Item:</u> (Closed book, five minutes.)
Outline a method of solution of the following problem:

A mathematics professor's car goes through a red traffic light at 45 miles per hour and continues along the straight street at the same speed. A police car, manned by a drop-out from the professor's calculus course, leaves the light in eager pursuit two seconds later, starting from at rest and accelerating at nine feet per second per second. How far down the straight street from the traffic light does the policeman overtake the professor?

14. Given the motion problem in 13, the student will, in five minutes, with no references, carry out the computations, making no more than two errors in arithmetic.

UNIT 4: REVIEW

This unit considers techniques of solving word problems, mathematical proofs and the place of analytic geometry and calculus in the history of mathematics.

Objectives

I. Goal: The student will broaden his repertoire of problem-solving techniques.

Objective: 1. Given a list of five word problems involving two or more concepts from this course, the student will, in one hour, with no references, outline the solutions of at least three.

Sample Word Problem: (Closed book, one hour.)

A Maniacal Drop-out caught a professor, and is laughing hysterically as he lowers the professor, bound and gagged, slowly into a cylindrical tank (radius of the circular bottom: six feet) partially filled with hydrochloric acid. If he's lowering the professor in a vertical position at the rate of half a foot a second, h is the depth of the acid at the side of the tank when the lowest extremity of the professor is x feet below the surface, and C(x) is the cross-sectional area of the professor at the surface at this time, find an expression for the rate of change of h.

II. Goal: The student will perfect his ability to analyze, synthesize and evaluate proofs.

Objective: 2. Analysis: Given a proof from the course textbook of a theorem from elementary differential calculus that involves three or more implications, the student will in 10 minutes, with no references other than the text, identify at least three implications, specifying for each the hypothesis and conclusion, these latter two with at least 80% completeness.

Sample Test Item: (Open book, 10 minutes.)

In the proof of the Mean-Value Theorem on pages 240-241 of the text, identify at least three implications, specifying for each the hypothesis and conclusion.

3. Synthesis: Given (without proof) an established theorem of elementary differential calculus, the proof of which is not included in the course textbook, the student will in 20 minutes, with the textbook as his only reference, write a proof of the theorem in which 80% of his implications are logically correct, assuming all results stated in the course textbook.
<u>Sample Test Item:</u> (Open book, 20 minutes.)
Prove the following theorem: If f is differentiable when $a < x < b$, with $f'(x) \geq 0$ for all x and $f'(x_o) > 0$ for a certain x_o, and $a < x_1 < x_o < x_2 < b$, then $f(x_1) < f(x_2)$.

4. Evaluation: Given a fellow student's paper written in hoped-for attainment of objective e above, the student will, with the course textbook as his only reference, ascertain in 30 minutes with at least 90% accuracy, whether or not the fellow student has indeed attained the objective.
<u>Sample Test Item:</u> (Open book, 30 minutes.)
Given a fellow student's proposed proof of the theorem in e, determine if his chain of implications <u>does</u> lead to the conclusion of the theorem and if 80% or more of his implications are logically correct.

III. <u>Goal:</u> The student will broaden his knowledge of the historical aspects of calculus.

<u>Objective:</u> 5. Given a list of three problems of calculus and analytic geometry, the student will, in five minutes, with no references, place at least four of them in the century in which they were first solved.
<u>Sample Test Item:</u> (Closed book, one minute.)
In what century was the problem of area under a parabola first solved? (3rd B.C., 6th A.D., 14th A.D., 19th A.D., or 20th, A.D.)

6. The student will, in two minutes, with no references, list three mathematicians who have attained the eminence in mathematics that Einstein attained in physics, with errors in name-spelling permissible.

CALCULUS AND GEOMETRY OBJECTIVES: SET # 3

The textbook for this course is <u>Calculus</u> by Walter Leighton, published by Allyn and Bacon, Inc., 1958.

Behavioral Objectives

In the following, general styles of functions are given because of the impossibility of designating all possible functions, indeed even of the infinite number of general functions.

I. The Derivative of a Function

1. The Concept of a Function

   Given a function of the style $f(x) = 1 - x + x^2$ the student will compute the value $f(b)$ for the given constant b.

2. The Graph of a Function

   Given a function of the style $f(x) = \frac{x^2}{x^2 - 4}$ the student will sketch the graph of the function.

3. Limits

   Given a function of the style $f(x) = \frac{x}{x^2 + 1}$ the student will compute the limit value of the function as x approaches some constant value as a limit.

4. The Derivative

Given a function of the style $f(x) = \frac{3x}{1-x}$ the student will compute the derivative of the function without the aid of theorems, using the definition of the derivative only.

5. Rates and Velocities

   Given a physical situation involving a linear function, the student by computing the derivative of the function and using standard physical formulas such as velocity, area, etc. will find the velocity, area, or some other physical measurement as asked.

6. Tangents; Slope of a Curve; Normals

   Given a function of the style $f(x) = \sqrt{x-2}$ the student will compute the equations of the tangent and normal lines to the curve of the function at a given point.

7. Maxima and Minima

   Given a function of the style $f(x) = \frac{x^2}{1+x}$ the student will find the maximum and minimum points of the curve of the function.

8. Continuous Functions

   Given a function of the style $\begin{cases} f(x) = x+1 & (x \neq 1) \\ f(1) = 2 \end{cases}$ the student will state whether or not the function is continuous at a given point.

II. Some Basic Formulas of Differentiation

1. Theorems on Limits

   Given a function of the style $f(x) = \sqrt{3x-2}$ the student will differentiate the function using the Δx- notation.

2. The Derivatives of Sums, Products, and Quotients

Given a function of the style $f(x) = \dfrac{x^{-1} - x^{-2}}{x^{-1} + x^{-2}}$ the student will differentiate the function using standard theorems.

3. The Differentiation of a Function of a Function

   Given a function of the style $f(x) = (x^2 + 5x)^{10}$ the student will differentiate the function using the function of a function concept.

4. The Differential

   Given a function of the style $f(x) = (1 + x^2)^{10}$ the student will compute $df(x)$.

5. Parametric Equations

   Given functions of the style $x = \dfrac{2t}{1+t^2}$, $y = \dfrac{1-t^2}{1+t^2}$ the student will sketch the curve.

III. Implicit Functions

1. The Differentiation of an Implicit Function

   Given a function of the style $x^2 + y^2 = 25$ the student will compute the equations of the tangent and normal lines to the curve of the function at a given point.

IV. Curve Tracing; Maxima and Minima

1. Elements of Curve Tracing

   Given a function of the style $y = x^2(x-3)$ the student will sketch the curve of the function.

2. Asymptotes

   Given a function of the style $y = \dfrac{(x-1)(x+1)^2}{(x-2)^2(x+2)}$ the student will sketch the curve of the function.

3. Slant Asymptotes

   Given a function of the style $y = \dfrac{(2x-1)^2}{x+2}$ the student will sketch the curve of the function, find

and draw in all asymptotes.

4. Graphs of Certain Other Algebraic Functions

   Given a function of the style $y^2 = \frac{-3x(x-3)^2}{(x+1)^2(x-2)^3}$ the student will sketch the curve of the function.

5. Maxima, Minima, and Points of Inflection

   Given a function of the style $y = x^3 - 3x + 2$ the student will find maxima, minima, and points of inflection.

V. The Definite Integral

1. The Definite Integral

   Given a definite integral of the style $\int_{-1}^{2} (x+2)\,dx$ the student will evaluate it without the use of theorems on integration, using the summation method.

2. Some Theorems on Definite Integrals

   Given a definite integral of the style $\int_{-1}^{1} (2x+x^5)\,dx$ the student will evaluate it using standard theorems.

3. The Fundamental Theorem of the Integral Calculus

   Given a definite integral of the style $\int_{0}^{2} (2x-3x^2)\,dx$ the student will evaluate it using standard theorems and the fundamental theorem.

4. The Area Under a Curve

   Given a curve $y = x^2$ the student will find the area under the curve between the lines $x=a$ and $x=b$.

5. On Formal Integration

   Given an integral of the style $\int (x-1)(2-x)\,dx$ the student will evaluate it.

6. Integration by Substitution

   Given an integral of the style $\int \sqrt{1-x^2}\, x\,dx$ the student will evaluate it by using an appropriate

substitution.

7. Volumes

Given a physical situation involving a simple geometrical figure, the student will use the integration process to find the volume of the figure.

VI. Trigonometric Functions

1. Definitions; Elementary Identities

Given a trigonometric identity the student will prove it using the elementary identities.

2. Radian Measure

Given a degree measure the student will express it in radian measure.

3. The Derivatives of the Trigonometric Functions

Given a function of the style $f(x) = \frac{1 - \sin 2x}{1 + \sin 2x}$ the student will differentiate the function using standard theorems.

4. Inverse Trigonometric Functions

Given a function of the style $y = \arcsin(\sin x)$ the student will find the equations of the tangent and normal lines to the curve of the function at a given point.

5. Integrals

Given an integral of the style $\int \cos 3x \, dx$ the student will evaluate it using standard theorems.

6. Polar Coordinates

Given a function of the style $\rho = \cos 3\theta$ the student will sketch the curve of the function.

7. Areas in Polar Coordinates

Given a curve of the style $\rho^2 = a^2 \cos \theta$ the

student will find the area enclosed by the curve.

VII. Logarithmic and Exponential Functions; Law of the Mean

1. The Logarithm

   Given a function of the style $f(x) = \log(x^2 - 2x + 2)$ the student will compute maxima, minima, and points of inflection of the curve of the function.

2. The Exponential Function

   Given a function of the style $f(x) = e^{\log x^2}$ the student will differentiate the function using standard theorems.

3. Some Related Integrals

   Given an integral of the style $\int \frac{\log x}{x} dx$ the student will evaluate it using standard theorems.

4. Hyperbolic Functions

   Given a function of the style $y = \tanh x$ the student will sketch the curve of the function.

5. L'Hospital's Rule

   Given a function of the style $f(x) = \frac{1 + \tan x}{4x - 3\pi}$ the student will find the limit value of the function as x approaches a given limit value, using L'Hôpital's rule.

VIII. Formal Integration

1. Trigonometric Integrals

   Given a trigonometric integral the student will evaluate it using standard theorems.

2. Trigonometric Substitutions

   Given an integral of the style $\int \frac{dx}{\sqrt{a^2+x^2}}$ the student will evaluate it using a trigonometric substitution and other standard theorems.

3. Partial Fractions

Given an integral of the style $\int \frac{2x+1}{(x+1)(x^2+1)}\,dx$ the student will evaluate it using theorems on partial fractions.

4. Rational Functions of the Trigonometric Functions

Given an integral of a rational trigonometric function the student will evaluate it using standard theorems.

5. Integration by Parts

Given an integral of the style $\int x \sin x \, dx$ the student will evaluate it using the method of integration by parts.

CALCULUS AND GEOMETRY OBJECTIVES: SET # 4

TEXT

Richard E. Johnson and Fred L. Kiokemeister. Calculus with Analytic Geometry. 2nd. ed. Boston: Allyn and Bacon, 1960.

UNIT 1: Topics from Algebra

Calculus, like algebra is concerned with the properties of numbers. However, unlike algebra it has to do with the concept of limit rather than such purely algebraic concepts as factorization, solving equations, etc. This unit is designedto refresh the students memory on certain prerequisite algebraic concepts and the introduction of certain topics which are of secondary importance to algebra but are of primary importance in calculus. A pretest will be given the secondclass meeting to determine where we should begin our discussion of algebra.

Objectives:

The student will display an ability to apply the techniques of intermediate algebra by solving 1) a list of linear equations for one unknown 2) several sets of simultaneous equations for two and three unknowns 3) factoring a list of polynomials 4) solving a set of quadradic equations. Performance will be demonstrated under standard exam conditions lasting the full second class meeting. 80% accuracy is expected for each of the four items.

The following objectives will be assessed on the first exam, closed book, one hour in length.

1. The student will define the intergers, the rational numbers, the irrational numbers, the complex numbers, absolute value, inequalities, coordinate lines, intervals, closed intervals, open intermals, open intervals. 90% accuracy is expected.

2. The student will restate these definitions verbally without the use of mathematical symbols. He may use graphs and illustrations. 70% ac-

curacy is expected.

3. The student will be able to solve a set of problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 2: Introduction to Analytic Geometry

Many applications of the calculus are found in the solution of geometric problems such as that of finding the area of circle, the length of curved lines, tangents to curved lines, or the volumes of solid figures. The basis of these applications lies in that part of mathematics known as analytic geometry in which numbers are used to determine the position of points, and equations are used to describe geometric figures.

Objectives:

The following objectives will be assessed on the first exam.

1. The student will define the following terms or state the following theorems: Graph of an equation, the distance formula, circle equation, slope of a lime, formula for the slope of a line, conditions for parallel or perpendicular lines. 80% accuracy is expected.

2. The student will restate these definations and theorems verbally (in writing) without the use of mathematical symbols. He may use graphs and illustrations. 70% accuracy is expected.

3. The student will be able to solve a set of problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 3: Functions

A general theory of the calculus is not possible without the use of functions. In fact, the word function with a strict mathematical meaning was introduced by Leibnitz, one of the discoverers of the calculus. Therefore, before getting into calculus proper, we should discuss functions.

Objectives:

The following objective will be assessed on the first exam, closed book, one hour in length.

1. The student will define the following terms or state the following theorems: function, domain, range, product functions, sum functions, and the difference between functions, increasing function, decreasing functions, monotone functions. 90% accuracy expected.

UNIT 4: Limits and Derivatives

In this unit we lay the foundation for the calculus. This foundation will consist of definations and basic theorems. The new concepts are limit, continuity, and derivative. The introduction of limits is the quantum jump that allows us to proceed from elementary mathematics such as algebra and geometry to higher mathematics - the calculus.

Objectives:

The following objectives will be assessed on the second exam, closed book, one hour in length.

1. The student will define the following terms, or state the following theorems: limit, continuity, the limit theorems (text designation 4.15, 4.16, 4.17, 4.18, 4.19, 4.20, 4.21, 4.22, 4.23), derivative, tangent line, continuity of a differentiable f'n theorm. 95% accuracy is required. (The student will find that this is one of the crucial objectives of the course. If he has difficulty with it he is advised to seek aid from the instructor.)
2. The student will restate the above definations and theorems verbally (in writing) without the use of mathematical symbols. He may use graphs and illustrations. 70% accuracy is expected.
3. The student will be able to solve a set of problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 5: Differentiation of Algebraic Functions

Algebraic functions are encountered very often in practice. In this unit we will develop a set of differentiation formulas that will allow us to find the derivative of every algebraic function.

Objectives:

The following objectives will be assessed on the second exam, closed book, one hour in length.

1. The student will be able to define the following terms or state the following theorems: sum formula, product and quotient formulas, the chain rule, and higher derivatives.

2. The student will state the above definitions and theorems verbally (in writing) without the use of mathematical symbols. He may use graphs and illustrations, 70% accuracy is expected.

3. The student will be able to solve a set of problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 6: Applications of the Derivative

We have already discussed one application of the derivative, namely the problem of finding tangent lines to the graph of a function. One of the common applications of the derivative concerns the best way to do something - maximum or minumum values of a function. The derivative is uniquely suited to this application. The derivative is also used to find the velocity and acceleration of moving objects. We will be concerned with these and other applications of the derivative in this unit.

Objectives:

The following objectives will be assessed on the second exam, closed book, one hour in length.

1. The student will define the following terms or state the following theorems: tangent line, extremum of a function, rules for finding extremum and inflection points of a function, Rolle's theorem, concave upward, concave downward, Mean Value Theorem, velocity, acceleration, antiderative.
2. The student will restate the above definitions and theorems verbally (in writing) without the use of mathematical symbols. He may use graphs and illustrations. 70% accuracy is expected.
3. The student will solve a set of problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 7: The Definite Integral

In this unit we introduce the other principle topic of the calculus, the integral. Certain additional concepts are introduced in this unit to lay the foundation for the integral. The theory of integration is the most elegant found in the calculus.

Objectives:

The following objectives will be assessed on the third exam, closed book, one hour in length.

1. The student will be able to define the following terms and state the following theorems: upper bound, lower bound, upper and lower integrals, theorems 8.9, 8.10, 8.11, 8.12, 8.13, definite integral, theorem 8.15, The Fundamental Theorem of the Calculus, change of variable transformation, sequences, Riemann sums, intermediate value theorem. 80% accuracy.

2. The student will be able to restate the above theorems and definations verbally (in writing) without the use of mathematical symbols. 70% accuracy is expected.

3. The student will solve problems similar to those assigned for homework. 70% accuracy is expected.

UNIT 8: Applications of the Integral

In this unit we will cover two of the common applications of the integral - the determination of the areas bounded by regions in a plane and the volume contained in solid figures.

Objectives:

The following objectives will be assessed on the third exam, closed book, one hour in length.

1. The student will find the area in a region bounded by algebraic functions. 80% accuracy is required.
2. The student will find the volume of solids formed by rotating algebraic functions about their axis. 40% accuracy is expected.

UNIT 9: Terminal Objectives

As stated in the course introduction, this course is designed to give the student an ability to perform the processes of differentiation and integration and to apply the processes to common problems. This unit is designed as a review of the past eight units and as preparation for assesment of the course objectives.

Objectives:

1. The student will apply the process of differentiation or integration to a list of 100 functions. Use of tables will be permitted in otherwise examination conditions without time limit. 90% accuracy is expected.

2. The student will solve a list of 20 common problems requiring the use of the processes of integration and/or differentiation. Use of tables will be permitted in otherwise examination conditions without time limit. 70% accuracy is expected.

CALCULUS AND GEOMETRY OBJECTIVES: SET # 5

Text: Calculus and Analytic Geometry, Thomas, George B. Addison Wesley, 3rd Edition.

UNIT I -- Rate of Change of a Function

I. General Objective - the student will become acquainted with the analytic geomentry of the straight line.

Specific Objective -- (1) the student will define in one sentence the term slope of a straight line.

(2) Given the graph of a line, he will calculate the slope of the line.

(3) Given the equation of a line, he will determine by analysis the slope of the line

(4) The student will state in one sentence the relationship between the slope of a given line and the slope of a line perpendicular to the given line.

(5) The student will derive the equation for a straight line in both the point-slope form and in the slope intercept form.

(6) The student will solve problems 1-24 in section 1-5 of the text.

UNIT I - Rate of Change of a Function

II. General Objective: The student will know the meaning of the term function and how to graph a function.

A. Specific Objective:

1. The student will define the following terms in one sentence for each item: variable, independent and dependent variables, domain of a variable, range of a variable, function.
2. Given a function, the student will be able to graph the function on a set of cartesian corrdinate axas.
3. The student will do problems 1, 2, 3, 4, & 5 in Section 1-6.

UNIT I - Rate of Change of a Function

III. General Objective: The student will know how to obtain the derivative of a function using the Δ process.

Specific Objective:

1. The student will distinguish between the secant line between any two points on a given curve and the tangent line at any point of the two points given on the curve.
2. The student will, in 2-3 sentences, explain the relationship between the slope of the secant line between any two points on the curve using the concept of the limiting value of the slope.
3. Using the Δ process, the student will evaluate the slope of a curve at any point.
4. The student will write the equation defining the derivative of a function.
5. The student will graph the function, using the value of the derivative to determine horizontal tangents to the curve.
6. The student will do problem 1, 3, 5, 7, 9, 11, 13, & 15 in section 1-7 & problems 1, 3, 5, 7, 9, 11, 18, & 20 in section 1-8

UNIT I - Rate of Change of a Function

IV. General Objective - the student will be able to solve velocity and rate problem

Specific I - The student will express the velocity of a moving body in terms of the defining expression of the derivative.

II - The student will express the time rate of change of a physical quanity (y) expressed as a function y f (n) in terms of the defining espression of the derivative.

III - The student will express the geometrical gate of change of a function in terms of the defining expression of the derivative.

IV - The student will solve problem 1, 3, 5, 7, 9, 10, section 1-9.

UNIT I - Rate of Change of a Function

V. General Objective - The student will know how to evaluate the limit of a function.

Specific Objective - (1) The student will in two to three sentences, give the definition of a limit.

(2) The student will in two to three sentences, state when a limit may or may not exist.

(3) The student will solve problems 1-7 section 1-10.

Unit II - Derivative of Algebraic Functions

I. General - The student will know the expressions for the derivative of the simple monomial & polynomial algebraic functions.

Specific I. The student will state and prove the formula for the derivative of the function $y =$ a constant, c.

Specific II. The student will state and derive the formula for the derivative of the function $y = x^n$ when n is any positive integer

Specific III. The student will state and derive the formula for the derivative of the function $y = c\,u$ where $u = f(x)$ and from it develop an expression for the derivative of $y = cx^n$

Specific IV. The student will state and derive the formula for the derivative of $y = u + v$ where u & v are differentiable functions of x

Specific V. The student will do problems 1-15, 16, 17, 19 and 22 in section 2-1.

II. General - The student will know the formulas for the derivatives of rational algebraic expressions.

Specific I. The student will state and derive the formula for the derivative of the product of two differentiable functions, i.e. $y = uv$ where $u = f(x)$, $v = g(x)$

Specific II. The student will state and derive the formula for the derivative of the quotient $y = \frac{u}{v}$ of two differentiable functions, u & v

Specific III. The student will state and derive the formula for the derivative with respect to x of u^n where n is a positive integer and u is a differentiable function of x

Specific IV. The student will do problems 1-15 Section 2-2.

III. General - The student will know how to obtain the derivative of algebraic functions in which y is an implicit function of x

Specific I. Given an equation in which y is expressed implicitly as a function of x, the student will be able to obtain $\frac{dy}{dx}$ of the function

Specific II. The student will be able to obtain the derivative of the function $y = \mu^{m/n}$ where μ is some differentiable function of x & m/n is the ratio of two integers and $n \neq 0$

Specific III. The student will do problems 1, 2, 3, 4, 6, 8, 10, 12, 25, 26, 30, 31 Section 2-3

IV. General -- The student will know how to estimate the increment of a function when the independent variable undergoes an increment

Specific I. The student will in his own words distinguish between the principal part and the remainder of an increment in a function and demonstrate these quantities in a graph

Specific II. The student will be able to approximate the value of a function to a given order of accuracy

Specific III. The student will do problems 1-5 Section 2-4

V. General - The student will know the chain rule in obtaining the derivative of functions.

Specific I. The student will define what is meant by the term "parametric equations" and will state when parametric equations occur.

Specific II. The student will state the chain rule for obtaining the derivative, $\frac{dy}{dx}$ of two functions, $x = f(t)$; $y = g(t)$

Specific III. Given a pair of parametric equations, y & x as functions of (t) the student will obtain the derivative $\frac{dy}{dx}$ by two methods

1. using the chain rule
2. elimination of the parameter, t and subsequent implicit differentiation

Specific IV. The student will do problems 1-4, and 6-10, Section 2-5

VI General - The student will know the meaning and application of the differential.

Specific I. The student will define the differential and give geometrical interpretations of dy & dx

Specific II. The student will know how to obtain the second derivative and the second differential of a function

Specific III. The student will state all of the formulae derived for the derivative of the algebraic function in terms of the differential

Specific IV. The student will do problems 1-8, 14-18, Section 2-7.

Unit III Applications of the Derivative

I General Objective: The student will know the significance of the first and second derivative with respect to curve tracing.

Specific Objectives:

1. Given $y = f(x)$ the student will be able to sketch the curve of the function by the following two procedures;

   (a) Obtain the first derivative, $f'(x)$ to determine horizontal tangent points, and (b) examine the roots of the equation $f'(x) = 0$ to determine the slope of the line between the horizontal tangents.

2. Given $y = f(x)$ the student will be able to sketch the curve of the function using the test of the second derivative to determine the shape of the concavity with their appropriate intervals.

3. The student will define what is meant by the term point of inflection.

4. Given $y = f(x)$ the student will determine the points of inflection.

5. The student will do problems 1-5 section 3-1 and problems 1,2,4,5,8,10 section 3-4.

UNIT III — Applications of the Derivative

II. General Objective — The student will know how to solve related rate problems.

Specific Objectives — (1) Given a geometrical functional relationship in which the variables are time related, the student will express these relationships in the form of an equation.

(2) The student will solve problems 3, 5, 6, 7, 8 section 3-2.

UNIT III - Applications of the Derivative

III. General Objective - The student will know how to solve maxima and minima problems.

Specific Objectives - (1) Given a description of some functional relationship, the student will write an equation for the quantity that is to be a maximum or a minimum.

(2) With the use of the tests involving the first and second derivatives, the student will determine those values of the function which are maximum or minimum.

(3) The student will volve problems 2-10 section 3-6.

UNIT III - Applications of the Derivative

IV. General Objective - The student will know the mean value theorem.

Specific Objectives - (1) In one to two sentences, the student will define the mean value theorem for a given function y = stp and is continuous for ac x cb.

(2) The student will solve problems 1-5, section 3-8.

Unit IV - Integration

I. General Objective - The student will know how to obtain the indefinite integral of a function.

Specific I - The student will explain in one sentence, the relationship between anti-differentiation and indefinite integration.

Specific II - The student will write the formulas for the following differential functions.

$$\int du$$

$$\int a\,du$$

$$\int (du + dv)$$

$$\int u^n du$$

Specific III - The student will solve problems 1-25 in section 4-2.

Unit IV - Integration

II. General Objective - The student will know the applications of the indefinite integral.

Specific I - Given a differential equation, the student will solve the equation determining the specific value of the constant of integration for prescribed initial conditions.

Specific II - Given the general statement of the motion of a body, the student will derive the equations for the velocity and position of the body as a function of time.

Specific III - The student will solve problems 1, 3, 5, 7, 9, 11 & 13 section 4-3.

III. General Objective - The student will know how to differentiate and integrate sine and cosine functions.

Specific I - Using the Δ process, the student will derive the expression for the derivative of the function, $y = \sin u$.

Specific II - The student will derive the expression for the derivative of cos u, using the trigonometric identity relating cos u to sin u and the derivative of sin u.

Specific III - The student will write the formulas for the following differential function.

$$\int \cos u \, du$$

$$\int \sin u \, du$$

Specific IV - The student will solve problems 21-36 and problems 40-60 in section 4-5.

IV. General Objective - The student will know how to obtain the definite integral of a function.

Specific I - Given a function $y = f(x)$, the student will define what is meant by the definite integral between the limits of a & b in terms of the indefinite integral.

V. General Objective - The student will know how [illegible] a curve between prescribed [illegible].

Specific I - Given a function $y = f(x)$ on the closed interval [illegible] the student will by the method of division [illegible] rectangular strips calculate an approximation [illegible] the area under the curve given by the function $y = f(x)$.

Specific II - The student will solve problems 1-9 section 4-6.

Specific III - The student will derive the relationship between the [illegible] $\Delta x \to 0$, and the definite integral $\int_a^b f(x)dx$.

Specific IV - The student will state the fundamental theorem of the integral calculus.

Specific V - The student will solve problems 1, 3, 5, 7, [illegible] 16, section 4-8 and problems 6-12 section 4-9.

UNIT V - Applications of the Definite Integral

I. **General Objective** - **The student will know how to calculate the area between two curves.**

Specific Objectives (1) **Given two functions, $y_1 = f_1(x)$, and $y_2 = f_2(x)$ the student will graph the two functions and set up the area for the differential element $(y_1 - y_2)\Delta x$.**

(2) **The student will then be able to write the expression for the total area between the two curves between assigned limits using the fundamental theorem of the integral calculus.**

(3) **The student will solve problems 2-8, section 5-2.**

UNIT V - Applications of the Definite Integral

II. General Objective - The student will know how to calculate distances traveled by a body when moving with a variable velocity.

Specific Objectives (1) Given the function $v = f(t)$, the student will solve the resulting differential equation for the distance traveled between assigned limits of t, using the fundamental theorem of the integral calculus.

(2) The student will solve problems 1-8, section 5-3.

UNIT V - Applications of the Definite Integral

III. General Objective - The student will know how to calculate volumns of solids produced by revolving areas about given axes.

Specific Objectives - (1) Given the function $y = f(x)$, the student will write the expression for the elemental volume consisting of a "slice" of radius ~~y~~ y and of thickness Δx.

(2) The student will then write the expression for the total volume between assigned limits using the fundamental theorem of the integral calculus.

(3) Given the function $y = f(x)$, the student will write the expression for the elemental volume consisting of a "shell" of radius y, and thickness Δy.

(4) The student will then write the expression for the total volume between assigned limits using the fundamental theorem of the integral calculus.

(5) The student will do problems 1-8, section 5-4.

UNIT V - Applications of the Definite Integral

IV. General Objective - The student will know how to calculate the arc length of a plane curve.

Specific Objectives - (1) The student will derive the expression for the arc length of a curve (y =) using the mean value theorem and the fundamental theorem of the integral calculus.

(2) Given a function in terms of parametric equations; x = and y = the student will calculate the length of an arc of the curve between assigned limits.

(3) The student will solve problems 1-3, section 5-6.

Unit V - Applications of the Definite Integral

V. General Objective - The student will know how to calculate the area of a surface of revolution.

Specific Objectives - (1) The student will derive the two expressions for surface area of generated by a function y = f(x) when rotated about the x or y axis, using the fundamental theorem of the integral calculus.

(2) The student will solve problems 2,3,4,5, and 6, section 5-7.

UNIT V - Applications of the Definite Integral

VI. General Objective - The student will know how to calculate for the center of mass of a body.

Specific Objectives - (1) The student will derive the equations for the center of mass of a system of discrete particles, using the principle of moments.

(2) The student will derive the equations for the center of mass of a body.

(3) The student will solve problems 1-5, section 5-1.

Unit V - Applications of the Definite Integral

VII - General Objective - The student will know how to calculate the work done by a variable force.

Specific I - The student will write the expression for work as defined in physics.

Specific II - The student will derive the equation for work done by a variable force over assigned limits.

Specific III- The student will solve problems 1-6 Section 5-13.